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Published in:

Journal of Physics: Conference Series

DOI:

[10.1088/1742-6596/451/1/012026](https://doi.org/10.1088/1742-6596/451/1/012026)

Publication date:

2013

Citation for published version (APA):

Slepyan, L. I., & Slepyan, V. I. (2013). Modeling of parametrically excited vibrating screen. *Journal of Physics: Conference Series*, 451(1), [012026]. <https://doi.org/10.1088/1742-6596/451/1/012026>

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2013 J. Phys.: Conf. Ser. 451 012026

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Modeling of parametrically excited vibrating screen

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Abstract. The dynamic model of a RR-based rectangular vibrating screen is considered as an initially stretched system of two equal masses connected by a linearly elastic spring. Due to the geometric nonlinearity longitudinal oscillations of the masses and lateral oscillations of the spring are coupled. Under certain conditions, when the masses are subjected by a self-equilibrated periodic longitudinal action, the parametric resonance arises which amplitude is bounded by the nonlinearity. The dynamic problem is reduced to a system of two ordinary nonlinear equations. An exact analytical solution is found existing under some conditions. In a general case, the dynamics of this system is considered numerically. The dissipation in this process is estimated. A comparative analysis of the dynamics of conventional and PR-based screeners is presented. Based on the analysis of the model a vibrating screen machine was designed, built and set up in LPMC. The machine operated as predicted.

1. Introduction

The idea to create a PR-based oscillating screen came to us in 2007 while discussing drawbacks of existing types of the machines. In 2009, Ukrainian patent was issued on the excitation method of an oscillation screen and the corresponding structure of the latter [1]. A mathematical model of the PR-based machine was developed. By means of numerical simulations parameters of its structure and setting were determined, which ensure effective sustained PR-oscillation mode. Note that while the parametric resonance was mainly considered as an undesirable phenomenon, some attempts were made to use it to obtain a greater response to a moderate excitation [2], also see [3-5]. In this paper, the following topics are briefly considered: The conventional vibrating screen versus the PR-based screen. The PR-based screen mathematical model. Free and forced oscillations of the system. An exact solution for a forced oscillation regime. Some results of numerical simulations and, lastly, Estimation of the dissipation based on analysis of the collision of a particle with some types of the sieve.

2. Conventional model with the lateral excitation of the sieve

We consider a plane problem for a flexible initially stretched sieve, $-l < x < l$, excited by lateral harmonic oscillations of its edges. In the linear approximation, the sieve dynamics is described by the wave equation for a string

$$T \frac{\partial^2 v(x, t)}{\partial x^2} - \rho \frac{\partial^2 v(x, t)}{\partial t^2} - \alpha \frac{\partial v(x, t)}{\partial t} = 0, \quad (1)$$



where $v(x, t)$ is the lateral displacement, T is the tensile force per unit length of the sieve, ϱ is the mass density per unit area and α is the respective viscosity number. For forced vibrations with frequency ω the oscillation amplitude is found to be distributed as

$$U(\xi) = U_0 \sqrt{\frac{\cosh^2(k_1 \xi) - \sin^2(k_2 \xi)}{\cosh^2(k_1) - \sin^2(k_2)}}, \quad k_{1,2} = \frac{1}{\sqrt{2}} \sqrt{\lambda_1^2 + \lambda_2^2 \mp \lambda_1}, \quad (2)$$

where $\lambda_1 = \varrho l^2 \omega^2 / T$, $\lambda_2 = \alpha l^2 \omega / T$, $\xi = x/l$.

Separation of wet granular materials, like sand or small-size gravel, meets a greater resistance due to the size-dependent surface effects. In the considered model, this reflects in increase of the viscosity number, that is, in the increase of parameter λ_2 , and in decrease of the oscillation amplitude. As a result, the conventional oscillation screens become inefficient. In contrast, the longitudinal oscillations of the sieve meet a very low inelastic resistance, and the PR excitation results in much greater amplitudes. The resonant regime corresponds to $\lambda_1 = \pi^2/4$. The results as the output-to-input ratio, $\Lambda_U = U(0)/U_0$ for the conventional screen and the lateral-to-longitudinal amplitude ratio for the PR-based screen, Λ_P , for $\lambda_2 = 1$ and $\lambda_2 = 2.5$ are as follows: $\Lambda_U = 3.137$, $\Lambda_P = 21.2$, $\Lambda_P/\Lambda_U = \mathbf{6.76}$ ($\lambda_2 = 1$); $\Lambda_U = 1.245$, $\Lambda_P = 13.3$, $\Lambda_P/\Lambda_U = \mathbf{10.7}$ ($\lambda_2 = 2.5$). This shows how the PR-based screen is efficient.

3. The PR-based screen model

The model considered here consists of two equal masses, M , connected by a spring of the mass density ϱ per unit length, the length $2l$ and the stiffness in tension $k/2$. Each of the masses is also connected with the rigid foundation by a side spring, which stiffness $\varkappa \ll k$, Fig. 1. The springs are initially stretched; the initial tensile force is denoted by T_0 . The symmetric motion is considered, in which the masses can oscillate moving only along the initial spring line (horizontally), while the central spring (the sieve) can oscillate in normal direction. The oscillations are excited by vibrators acting synchronically on the left and the right masses in opposite horizontal directions. The amplitude and frequency of the action are denoted by P and ω , respectively. The lateral oscillation amplitude is assumed to be much less than the spring length. Also it is assumed that the inequalities $c = \sqrt{kl/\varrho} \gg \omega l$, $M \gg \varrho l$ are valid. This allows us to assume that the tensile force in spring, T , is independent of the coordinate x , whereas the spring inertia can be taken into account only in the lateral motion. So, if there is no dissipation the main natural longitudinal and lateral frequencies for small amplitudes are $\Omega_L = \sqrt{k/M}$, $\Omega_T = \pi/(2l)\sqrt{T_0/\varrho}$.

Under oscillations of the sieve the granular material separated mainly hangs at a distance from the sieve. We assume that the material-sieve interaction results in a force equal to the material weight and in dissipation, which is presented as linear viscosity.

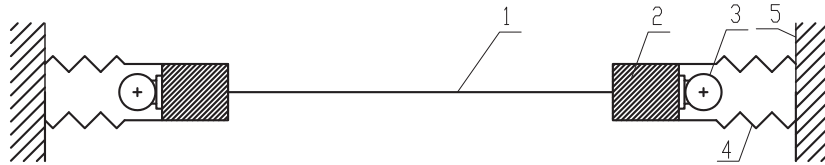


Figure 1. The PR-based oscillating screen model: the spring (the sieve) - 1, the edge mass - 2, the vibrator - 3, the side spring - 4 and the foundation - 5.

If ω is close enough to $2\Omega_T$ and P is large enough, the parametric resonance arises under which the amplitude of the lateral oscillations is bounded by the geometric nonlinearity. We

based on nonlinear dynamic equations with respect to the longitudinal and lateral oscillations

$$\begin{aligned} M \frac{d^2 u(t)}{dt^2} + \beta \frac{du(t)}{dt} + T_1(t) &= P \cos \omega t, \\ \varrho \frac{\partial^2 v(x, t)}{\partial t^2} + \alpha \frac{\partial v(x, t)}{\partial t} - T(t) \frac{\partial^2 v(x, t)}{\partial x^2} &= q(x, t), \end{aligned} \quad (3)$$

which are coupled by the tensile force, $T_1(t)$, depending on both the displacement of the masses, $\pm u(t)$ ($x = \pm l$) and the displacement of the spring, $v(x, t)$

$$T(t) = T_0 + T_1(t), \quad T_1(t) = k \left[u(t) + \frac{1}{2} \left(\int_0^l \sqrt{1 + (v'(x, t))^2} dx - l \right) \right], \quad (4)$$

where $v'(x, t) = \partial v(x, t) / \partial x$. In addition, in Eq. (3), α and β are the viscosity numbers and the averaged value of q is the treated material weight per unit length. Expression (4) is valid, however, if it defines a nonnegative tensile force; otherwise, $T(t) = 0$.

The left side of the second equation in (3) with the boundary conditions, $v(\pm l, t) = 0$, admits the variables separation. For the main term of the series, $v(x, t) = w(t) \cos(\pi x / 2l)$ we obtain

$$\begin{aligned} \varrho \frac{d^2 w(t)}{dt^2} + \alpha \frac{dw(t)}{dt} + \frac{\pi^2}{4l^2} T(t) w(t) &= q_0(t) = \frac{1}{l} \int_{-l}^l q(x, t) \cos \frac{\pi x}{2l} dx, \\ M \frac{d^2 u(t)}{dt^2} + \beta \frac{du(t)}{dt} + T_1(t) &= P \cos \omega t, \quad T_1(t) = k \left(u(t) + \frac{\pi^2}{16l} w^2(t) \right). \end{aligned} \quad (5)$$

4. Free oscillations

The total energy of the oscillations as the sum of the kinetic and potential energies is

$$\mathcal{E} = M[du(t)/dt]^2 + \frac{1}{2} \varrho l [dw(t)/dt]^2 + T_0 \frac{\pi^2}{8l} w^2(t) + \frac{1}{k} T_1^2. \quad (6)$$

It can be considered consisting of longitudinal and lateral parts as follows (note that this separation is in a sense conditional) :

$$\begin{aligned} \mathcal{E} &= \mathcal{E}_u + \mathcal{E}_w, \\ \mathcal{E}_u &= M[du(t)/dt]^2 + \frac{1}{k} T_1^2(t) \quad (\text{longitudinal part}), \\ \mathcal{E}_w &= \frac{1}{2} \varrho l [dw(t)/dt]^2 + \frac{\pi^2}{8l} T_0 w^2(t) \quad (\text{transversal part}). \end{aligned} \quad (7)$$

From this and the dynamic equations with $P = q = \alpha = \beta = 0$ we find the energy exchange rate between the longitudinal and transversal parts

$$\frac{d\mathcal{E}_u}{dt} = -\frac{d\mathcal{E}_w}{dt} = \frac{\pi^2}{4l} T_1 w(t) \frac{dw(t)}{dt}. \quad (8)$$

Another system of two-mode nonlinear oscillations with the PR-related ratio of frequencies, a spring pendulum, was considered long ago in [6] (also see [7]) and then in [8] assuming the angle amplitude small enough. In some respects, this system is similar to the oscillating screen, and we briefly consider it. The pendulum model consists of the point mass, M , connected to a fixed point by a massless spring of stiffness k . Let the initial spring length be l_0 , $l(t) = l_0 + u(t)$

and $\alpha(t)$ be the angle ($\alpha = 0$ is statics). In the exact formulation, with no restrictions on values of the angle and the spring elongation, the energy is

$$\mathcal{E} = \Pi + \mathcal{K}, \quad \Pi = \frac{1}{2}ku(t)^2 + Mgl(t)[1 - \cos \alpha(t)], \quad \mathcal{K} = \frac{1}{2}M[\dot{u}^2(t) + l^2(t)\dot{\alpha}^2(t)], \quad (9)$$

where g is the gravity acceleration. The dynamic equations follow from (6) as

$$\begin{aligned} M\ddot{u}(t) + ku(t) + Mg[1 - \cos \alpha(t)] - Ml(t)\dot{\alpha}^2(t) &= 0, \\ l(t)\ddot{\alpha} + g \sin \alpha(t) + 2\dot{l}(t)\dot{\alpha}(t) &= 0. \end{aligned} \quad (10)$$

We define the energy of longitudinal and tangential oscillations to be

$$\mathcal{E}_u = \frac{1}{2}ku(t)^2 + \frac{1}{2}M\dot{u}^2(t), \quad \mathcal{E}_\alpha = Mgl(t)[1 - \cos \alpha(t)] + \frac{1}{2}Ml^2(t)\dot{\alpha}^2(t), \quad (11)$$

respectively. Using the dynamic equations (10) we find the rate of the energy exchange between these parts of the total energy

$$\dot{\mathcal{E}}_u = -\dot{\mathcal{E}}_\alpha = [l(t)\dot{\alpha}^2(t) - g(1 - \cos \alpha(t))] M\dot{u}(t). \quad (12)$$

5. Forced oscillations

Consider equations (5) with $\alpha = q = 0$. There exists a periodic solution, in which the tensile force preserves its initial value, $T_1(t) \equiv 0$

$$\begin{aligned} u(t) &= -\frac{P}{\omega(M^2\omega^2 + \beta^2)} \left(M\omega \cos \omega t - \beta \sin \omega t + \sqrt{M^2\omega^2 + \beta^2} \right), \\ w(t) &= A \cos \frac{\omega t}{2} + B \sin \frac{\omega t}{2}, \quad T_1(t) = 0, \\ A &= \sqrt{\left(\sqrt{M^2\omega^2 + \beta^2} + M\omega \right) \psi}, \quad B = \sqrt{\left(\sqrt{M^2\omega^2 + \beta^2} - M\omega \right) \psi}, \\ \psi &= \frac{16lP}{\pi^2\omega(M^2\omega^2 + \beta^2)}, \quad (-u)_{\max} = \frac{2P}{\omega\sqrt{M^2\omega^2 + \beta^2}}, \quad w_{\max} = \sqrt{A^2 + B^2} = \frac{4}{\pi}\sqrt{-lu_{\max}}. \end{aligned} \quad (13)$$

Numerical calculations based on the dynamic equations (5) evidence that, in some range of the parameters, this solution is stable. It corresponds to the established regime for transient solutions with $T_1(0) \neq 0$.

For some other ranges of the parameters, including those with $\alpha \neq 0, q \neq 0$, other stable-oscillation regimes are revealed. An example is presented in Fig. 2

6. Dissipation

To estimate the energy consumption in screening we consider the collision of a particle with the sieve. The latter is represented in two models: an infinite elastic beam (the sieve consists of series of initially stretched parallel beams connected by weak links) and an elastic plate model. Using integral transforms, from the beam equation

$$Dw^{IV}(x, t) - Tw''(x, t) + \varrho\ddot{w}(x, t) = -m\ddot{w}(0, t)\delta(x)H(t), \quad (14)$$

where D is the bending stiffness and m is the particle mass, we have found the acceleration

$$\ddot{w}(\tau) = -\frac{2v_0a(s_0 + 1)}{a + 2\sqrt{s_0 + 1}}e^{s_0\tau} + \frac{v_0a}{\pi} \int_{-\infty}^{-1} \frac{s\sqrt{-s-1}}{s^2 - a^2(s+1)} e^{s\tau} ds, \quad -1 < s_0 < 0, a > 0, \quad (15)$$

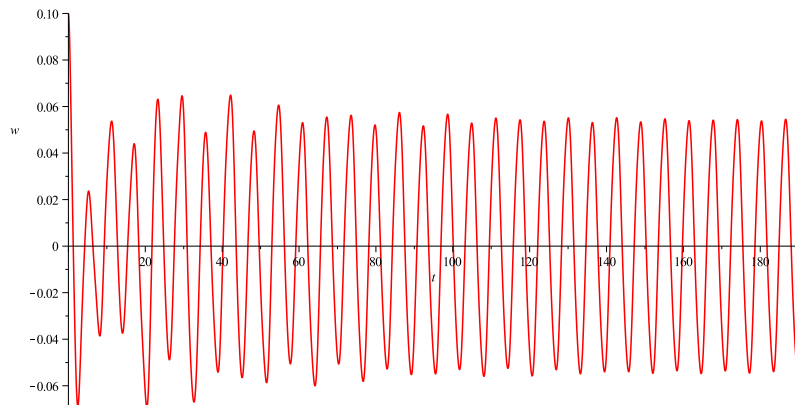


Figure 2. Forced PR-oscillations. The transversal oscillation at the center, $w(t)$; $M = 100, k = 250, T_0 = l = \varrho = 1, \alpha = 0.3, \beta = 0.05; w(0) = 0.1, u(0) = \dot{u}(0) = \dot{w}(0) = 0$.

where v_0 is the collision speed, τ is a nondimensional time. It can be seen that for $0 < t < \infty$ both terms are negative. This evidences that the collision is perfectly inelastic with zero restoration coefficient.

As the 2D model consider an infinite elastic plate stretched in x -direction and possessing the bending stiffness only corresponding to flexure in the (y, z) -plane. The dynamic equation is

$$\left(D \frac{\partial^4}{\partial y^4} - T \frac{\partial^2}{\partial x^2} + \varrho \frac{\partial^2}{\partial t^2} \right) w(x, y, t) = -m \frac{\partial^2 w(0, 0, t)}{\partial t^2} \delta(x) \delta(y) H(t), \quad (16)$$

From this equation the acceleration and the speed are found in the form

$$\ddot{w}(\tau) = v_0 b^{2/3} W(\tau), \quad \dot{w}(\tau) = v_0 \left(1 + \int_0^\tau W(x) dx \right) \quad (b > 0),$$

$$W(\tau) = -\frac{4}{3} \exp\left(-\frac{1}{2}\tau\right) \cos\left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\tau\right) + \frac{1}{\pi} \int \frac{x^{3/2}}{x^3 + 1} \exp(-x\tau) dx. \quad (17)$$

It can be seen that, in this model, the speed and the nondimensional acceleration have unique expressions, which are independent of the plate and particle parameters. In this 2D case, the restoration coefficient is equal to 0.303 (see Fig. 3), that is, only 9% of the kinetic energy of the collision remains. Taking into account other possible factors of inelasticity, it can be concluded that the energy of perfectly inelastic collision represents the lower bound of the dissipation.

7. Conclusions

Based on the above considerations the dynamic equations were developed to reflect additional factors important for the PR-based screen setting. In particular, this concerns the sieve-material inelastic interaction, where the collisions were assumed to take place only under upward directed speed of the screen. Also note that in the vicinities of the end masses, where the sieve curvature is considerably large, the bending stiffness must be taken into account. This is important for the estimation of the sieve strength under the high-amplitude oscillations. The corresponding edge-effect solution was constructed to satisfy the clamped boundary condition end and to correspond to the mass-string solution 'at infinity'. With respect to the prospective research note that the exact analytical solution presented here is valid under certain conditions. In a more general case, analytical results can be obtained using harmonic analysis, that we are going to do.

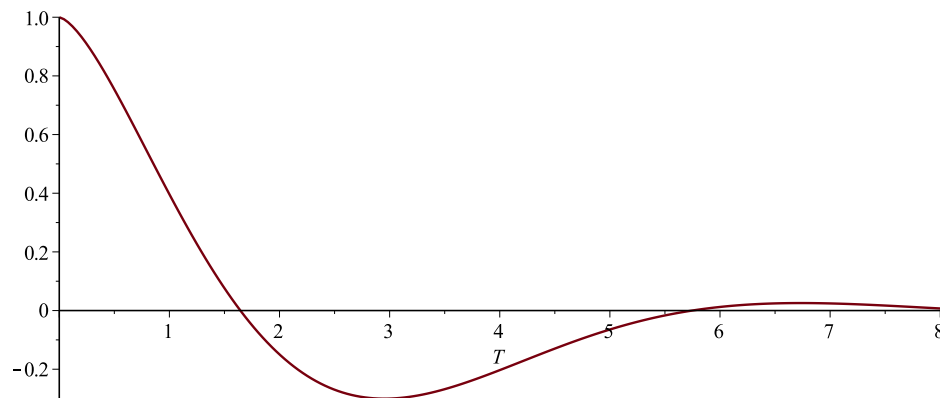


Figure 3. Collision of a particle with a plate: The nondimensional speed, $\dot{w}(\tau)/v_0$, based on (17).

Acknowledgments

We are thankful for the support provided by FP7-People-2011-IAPP European Union Grant No. 284544.

References

- [1] Slepyan V I, Loginov I G and Slepyan L I 2009 The method of resonance excitation of a vibrating sieve and the vibrating screen for its implementation *Ukrainian patent on invention* No. 87369
- [2] Rhoads J F and Shaw S W 2005 Tunable microelectromechanical filters that exploit parametric resonance *J. Vib. Acoust.* **127** 423-430
- [3] Krylov S, Harari I and Cohen Y 2005 Stabilization of electrostatically actuated microstructures using parametric excitation *J. Micromech. Microeng* **15** 1188-1204
- [4] Krylov S, Gerson Y, Nachmias T and Keren U 2010 Excitation of large-amplitude parametric resonance by the mechanical stiffness modulation of a microstructure *J. Micromech. Microeng* **20** 015041
- [5] Plat H and Bucher I 2013 Optimizing parametric oscillators with tunable boundary conditions *J. Sound Vib.* **332** 487493
- [6] Vitt A A and Gorelik G S 1933 *Zh. Tekh. Fiz.* **3** 294
- [7] Gaponov-Grekhov A V and Rabinovich M I 1992 *Nonlinearities in action* (Berlin:Springer)
- [8] Lai H M 1984 On the recurrence phenomenon of a resonant spring pendulum *Am. J. Phys.* **52**(3) 219-223